QUANTUM DRIVEN BOUNCE OF THE FUTURE UNIVERSE

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Abstract

It is demonstrated that due to back-reaction of quantum effects, expansion of the universe stops at its maximum and takes a turn around. Later on, it contracts to a very small size in finite future time. This phenomenon is followed by a "bounce" with re-birth of an exponentially expanding non-singular universe.

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Astrophysical data [1, 2] show cosmic acceleration in the current universe, which is fuelled by dark energy (DE) violating strong or weak energy condition (SEC or WEC). If DE mimics quintessence, the equation of state parameter (EOSP) w > -1 and SEC is violated . In the case of phantom (super-quintessence) DE, w < -1 and WEC is violated. It is found that when w< -1, DE leads the cosmological model having "big- rip singularity" and ending in cosmic doomsday with infinite energy density [3]. A comprehensive review on DE and accelerating universe is available in [4]. In contrast to quintessence model, phantom model shows growing curvature with time and in models with "big-rip singularity" at finite time t_s with t being the cosmic time, curvature invariants (containing high powers of curvature) grow very strong, when $|t_s - t|$ is sufficiently small. Quantum gravity suggests that quantum corrections to matter fields like scalars, spinors and vectors, in curved space-time, depend upon curvature. It implies dominance of quantum gavity near $t = t_s$. It is analogous to the early universe model, where curvature invariants are very strong and cause effective role of quantum corrections. This fact was pointed out in [5, 6], where it is demonstrated that singularity may be avoided or made milder using quantum corrections near *future singularity time* t_s.

Like [6], here also, scale factor of the universe is assumed in the beginning. Further, it is probed what type of conformal scalar (quintessence or phantom) support this scale factor (the assumed one) for being the solution of Friedmann equations. Like [5, 6], here also, quantum gravity effects are investigated when t approaches t_s. In [6], a possible escape from cosmic doomsday at $t = t_s$ is discussed using quantum corrections. Here, the approach is more elegant and a cosmic

scenario is obtained, where expansion stops at $t = t_m$ very near to t_s. As a result, it takes a turn around and contracts for a small period $(t_s - t_m)$. Subsequently, at $t = t_s$, cosmic bounce takes place and universe expands exponentially for $t > t_s$. This scenario emerges due to quantum effects during the period $(t_s - t_m)$. Thus, here, singularity is not made milder , but it is completely avoided in contrast to results in [5, 6]. In the present paper, this scenario is obtained for (i) general relativity (GR) model, (ii) Randall- Sundrum II (RSII) model and (iii) Gauuss-Bonnet(GB) model.

It is found that the scale factor reaches its maximum in finite time t_m, when scalar is phantom (super-quintessence) in GR and RSII models. But, in GB-model, similar situation is obtained for both quintessence conformal scalar and conformal phantom subject to different conditions. In [7] also, conformal phantom scalar, in FRW universe, had been considered in a different context. Here, an escape from 'future singularity' in phantom model is explored using quantum gravity effects of conformal phantom scalars, whereas, in [7], non-integrability of hamiltonian system of these fields is discussed using Ziglin theorems on integrability. Natural units ($\hbar = c = 1$) are used with $\hbar$ and c having their usual meaning. GeV is the fundamental unit and cosmic time t is measured in GeV^{-1}.

General Relativity model of the future universe

(a) Classical approach

According to experimental probes [8], our universe is spatially flat with growing scale factor $a(t)$, given by the distance function

$$dS^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2] \tag{1}$$

such that $\ddot{a} = \frac{d^2a}{dt^2} > 0$.

DE density ρ and pressure p are given by Friedmann equations

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \tag{2a}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{2b}$$

where $G = M_P^{-2}$ (M_P is the Planck's mass).

Here, source of the dark energy is conformal scalar $\phi(x)$ given by the action

$$S_\phi = \int d^4x\sqrt{-g}\frac{1}{2}\Big[\omega^2 g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{6}R\phi^2\Big], \tag{3a}$$

where $\omega^2 = \pm 1$ for quintessence and phantom (super-quintessence) scalars respectively.

This action yields the Klein-Gordon equation

$$\omega^2\ddot{\phi} + 3\omega^2 H\dot{\phi} + (\dot{H} + 2H^2)\phi = 0, \tag{3b}$$

where $R = 6(\dot{H} + 2H^2)$ and $\phi = \phi(t)$ due to homogeneity of the space-time (1).

The energy density ρ and pressure p are obtined from the action (3a) as

$$\rho = \frac{1}{2}\omega^2\dot{\phi}^2 + H\phi\dot{\phi} + \frac{1}{2}H^2\phi^2 \tag{3c}$$

and

$$p = \frac{1}{2}\omega^2\dot{\phi}^2 - \frac{1}{3}(\phi\ddot{\phi} + \dot{\phi}^2 + 3H\phi\dot{\phi}) - \frac{1}{6}(2\dot{H} + 3H^2)\phi^2. \tag{3d}$$

The conservation equation is

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{3e}$$

It is noted that the Klein-Gordon equation (3b) can be derived connecting eqs.(3c), (3d) and (3e). So, eqs.(3b) and (3e) are not independent implying that it is enough to use either of these two equations. Now, we are left with three equations (2a), (2b) and (3b) as well as (3c) and (3d) for ρ and p respectively. Connecting (2a) and (3c), it is obtained that

$$\dot{\phi} = H[-\omega^2\phi \pm \sqrt{\frac{3\omega^2}{4\pi G} + \phi^2(1-\omega^2)}]. \tag{4a}$$

This equation shows that when $\omega^2 = -1, \phi^2 \geq 3/8\pi G$. Using eqs.(3b), (3c), (3d) and (4a) in eq.(2b), it is obtained that

$$\aleph\phi^4 + 2\vartheta\phi^2 + \varkappa = 0 \tag{4b}$$

with

$$\aleph = [(6\omega^2 - 5)H^2 + (\omega^2 - 1)\dot{H}]^2 - 4(1-\omega^2)(\frac{3}{2}\omega^2 - 1)H^2, \tag{4c}$$

$$2\vartheta = \frac{3}{2\pi G}[\dot{H} + (3-\omega^2)H^2](6\omega^2-5)H^2 + (\omega^2-1)\dot{H}] - \frac{3}{\pi G}\omega^2(\frac{3}{2}\omega^2 - 1)H^2 \tag{4d}$$

and

$$\varkappa = \Big[\frac{3}{4\pi G}\Big(\dot{H} + (3-\omega^2)H^2\Big)\Big]^2. \tag{4e}$$

Eq.(4b) is obtained writing $\ddot{a}/a = \dot{H} + H^2$ as well as using $\dot{\phi}, \ddot{\phi}$ in terms of ϕ, H and $\dot{H}$. This equation yields

$$\phi^2 = \frac{-\vartheta \pm \sqrt{\vartheta^2 - \aleph\varkappa}}{\aleph}. \tag{4f}$$

Thus it is obtained that an $a(t)$ will satisfy eqs.(2a) and (2b), if conformal scalar ϕ obeys eqs.(4a) and (4b)[9]. Now, we can use conditions (4a) and (4b) to examine an $a(t)$ for being solutions of Friedmann equations (2a) and (2b) with conformal scalar ϕ as a source of DE.

For the time period $t_0 \leq t \leq t_m$, the form of $a(t)$ is assumed to be

$$a(t) = A + B(M_P t)^r + |M_P(t_s - t)|^{-q}, \tag{5a}$$

where $r > 1$ to get $\ddot{a}(t) > 0$, $q > 0, A$ and B are arbitrary constants. The classical approach shows that the scale factor (5a) has big-rip singularity at $t = t_s$ [10]. In the following section, it is shown that an escape from this catastrophic situation is possible using quantum gravity effects.

Now, we explore whether ϕ mimics quintessence or phantom, when $a(t)$ (given by eq.(5a)) satisfies eqs.(2a) and (2b) as well as acquires its maximum at time $t = t_m$. It turns out to explore ϕ, when $a(t)$ satisfies eqs.(4a) and (4b) for $t_0 \leq t \leq t_m$ with $a(t_m)$ being the maximum. With $a_0 = a(t_0)(t_0$ being the present time) A is obtained as

$$A = a_0 - B(M_P t_0)^r - \Big|M_P(t_s - t_0)\Big|^{-q}. \tag{5b}$$

$a(t)$, given by eq.(5a), expands to its maximum by the time $t = t_m$, if

$$\dot{a}(t_m) = rBM_P^r t_m^{(r-1)} + qM_P^{-q}|t_s - t_m|^{(-q-1)} = 0$$

implying

$$B = -\frac{qM_P^{(-q-r)}|t_s - t_m|^{(-q-1)}}{rt_m^{(r-1)}}. \tag{6}$$

Thus ,$a(t)$ from eq.(5a), looks like

$$a(t) = a_0 - \frac{q}{r}\frac{M_P^{-q}}{t_m^{(r-1)}}|t_s - t_m|^{(-q-1)}(t^r - t_0^r) + M_P^{-q}|t_s - t|^{-q} - M_P^{-q}|t_s - t_0|^{-q} \tag{7}$$

for $t_0 \leq t \leq t_m$.

Moreover,

$$\dot{a}(t) = qM_P^{-q}|t_s - t_m|^{(-q-1)}\Big[-\Big(t/t_m\Big)^{(r-1)} + \Big(|t_s - t|/|t_s - t_m|\Big)^{(-q-1)}\Big] \tag{8}$$

and

$$\ddot{a}(t) = qM_P^{-q}|t_s - t_m|^{(-q-1)}\Big[-\frac{(r-1)}{t}\Big(t/t_m\Big)^{(r-1)} + \frac{(q+1)}{|t_s - t|}\Big(|t_s - t|/|t_s - t_m|\Big)^{(-q-1)}\Big]. \tag{9}$$

These equations show $\dot{a}(t) > 0$ and $\ddot{a}(t) > 0$ for $t_0 < t < t_m$ and $q > 0, r > 1$ as

$$\frac{1}{|t_s - t|} > \frac{1}{t}. \tag{10}$$

Eqs.(8) and (9) yield that, at $t = t_m, H = 0$ and $\dot{H} \neq 0$.

Now, we have two cases (i) $\omega^2 = 1$ and (ii) $\omega^2 = -1$. Eq.(4f) yields ϕ^2 being *indeterminate* for $\omega^2 = 1$, when $H = 0$ and $\dot{H} \neq 0$ at $t = t_m$. But, for $\omega^2 = -1$

$$\phi^2(t_m) = \frac{3}{8\pi G} \tag{11}$$

for $H(t_m) = 0$ and $\dot{H}(t_m) \neq 0$.

It means that the scale factor $a(t)$, given by eqs.(5a), (5b) and (6) or eq.(7), is not a solution of eqs.(2a) and (2b) for the time period $t_0 \leq t \leq t_m$, if ϕ is a quintessence scalar, but it is a solution of eqs.(2a) and (2b) for the time period $t_0 \leq t \leq t_m$ for ϕ being a phantom scalar characterized by $\omega^2 = -1$.

This analysis suggests that if universe is dominated by quintessence dark energy, expansion will not stop at its maximum. So, in this model (GR model), only the case $\omega^2 = -1$ is investigated.

Quantum effects and bounce near t_s

In the scenario discussed above, $\dot{a}(t_m) = 0$, which means that either ρ in eq.(2a) vanishes at $t = t_m$ or there should be a term $\rho_{\rm corr}$ on the r.h.s. of the equation (2a), which remains ineffective during the time interval $t_0 < t < t_m$, but at $t = t_m$ it makes the effective energy density $\rho_{\rm eff} = \rho + \rho_{\rm corr} = 0$.

Using eqs.(4a), (5a,b) and (6) in eq.(3c), it is found that ρ increases with time during the interval $t_0 \leq t \leq t_m$. So, only second possibility may be valid as $\rho(t_m) \neq 0$. This argument modifies the Friedmann equation (2a) as

$$\Big(\frac{\dot{a}}{a}\Big)^2 = \frac{8\pi G}{3}(\rho + \rho_{\rm corr}) \qquad (12a)$$

and eq.(2b) as

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \rho_{\rm corr} + 3p + 3p_{\rm corr}), \qquad (12b)$$

such that for $t < t_m, \rho_{\rm corr} \approx 0$ and at $t = t_m, \rho = -\rho_{\rm corr}$. In what follows, we explore $\rho_{\rm corr}$ satisfying these requirements.

As it is found in eq.(10) that t_m also satisfies the inequality $|t_s - t_m| < 1$. So, when $t_s \gtrsim t \geq t_m$, curvature becomes very strong and quantum gravity becomes effective [5, 6, 11]. Conformal anamoly for scalars, due to quantum effects, yield correction to ρ and p as

$$\rho_A = \frac{N}{60(4\pi)^2}H^4 - \frac{N}{180(4\pi)^2}\{-6H\ddot{H} - 18H^2\dot{H} + 3\dot{H}^2\} \qquad (13)$$

and

$$p_A = -\frac{N}{360(4\pi)^2}(6H^4 + 8H^2\dot{H}) - \frac{N}{180(4\pi)^2}\{2\frac{d\ddot{H}}{dt} + 12H\dot{H} + 18H^2\dot{H} + 9\dot{H}^2\}, \qquad (14)$$

where N is the number of scalars.

Now, connecting eqs.(5a), (5b), (13) and (14), for $t_0 < t < t_m$, it is obtained that

$$\rho_A = -\frac{NqM_P^{-2q}}{60(4\pi)^2}|t_s - t|^{(-2q-4)}\Big[(q+1)$$
$$-(r-1)\frac{|t_s - t_m|^{(-q-1)}}{t}\Big(t/t_m\Big)^{(r-1)}|t_s - t|(q+2)\Big]^2. \tag{15}$$

This equation shows that for $t < t_m, \rho_A \approx 0$. But at $t = t_m$,

$$\rho_A(t_m) = -\frac{NqM_P^{-2q}}{60(4\pi)^2}|t_s - t_m|^{(-2q-4)}\Big[(q+1)$$
$$-(r-1)\frac{|t_s - t_m|^{-q}}{t_m}(q+2)\Big]^2. \tag{16}$$

Thus, we find that ρ_A satisfies the criteria required by $\rho_{\rm corr}$. So, it is set that $\rho_{\rm corr} = \rho_A$.

As $H(t_m) = 0$, eq.(13) yield $\rho(t_m) + \rho_A(t_m) = 0$, so

$$\rho(t_m) = \frac{NqM_P^{-2q}}{60(4\pi)^2}|t_s - t_m|^{(-2q-4)}\Big[(q+1)$$
$$-(r-1)\frac{|t_s - t_m|^{-q}}{t_m}(q+2)\Big]^2. \tag{17}$$

Thus it is obtained that DE density grows with time . Moreover, the correction term which is negligible for $t < t_m$ becomes very strong at $t = t_m$. As $\rho_A < 0, \rho_{\rm eff}(t_m) = \rho(t_m) + \rho_A(t_m) = 0$, but for $t < t_m$, $\rho_{\rm eff}(t_m) = \rho$. So, it is quantum effect, which stops expansion of the universe at $t = t_m$.

For the modified Friedmann equations (12a) and (12b), solution is taken as

$$a(t) = exp[B'(M_P t)^\alpha + C' M_P^\beta (t - t_m)^\beta + M_P^\gamma |t_s - t|^\gamma], \tag{18a}$$

when $t > t_m$. Moreover, $a(t)$ should be continuous at $t = t_m$, which is given by the condition $\lim_{\epsilon\to 0}\dot{a}(t_m + \epsilon) = \lim_{\epsilon\to 0}\dot{a}(t_m - \epsilon)$. Using this

condition for $a(t)$, given by eq.(5a) (when $t < t_m$) and $a(t)$, given by eq.(18a) (when $t > t_m$), it is obtained that

$$\alpha B' M_P^\alpha t_m^{(\alpha-1)} - \gamma M_P^\gamma |t_s - t_m|^{(\gamma-1)} = 0,$$

yielding

$$B' = \frac{\gamma M_P^\gamma |t_s - t_m|^{(\gamma-1)}}{\alpha M_P^\alpha t_m^{(\alpha-1)}}. \tag{18b}$$

For $t > t_m$, eq.(18a) yields

$$\begin{aligned}
H &= \alpha B' M_P^\alpha t^{(\alpha-1)} + \beta C' M_P^\beta (t - t_m)^{(\beta-1)} - \gamma M_P^\gamma |t_s - t|^{(\gamma-1)}, \\
\dot{H} &= \alpha(\alpha-1) B' M_P^\alpha t^{(\alpha-2)} + \beta(\beta-1) C' M_P^\beta (t - t_m)^{(\beta-2)} \\
&\quad + \gamma(\gamma-1) M_P^\gamma |t_s - t|^{(\gamma-2)}, \\
\ddot{H} &= \alpha(\alpha-1)(\alpha-2) B' M_P^\alpha t^{(\alpha-3)} + \beta(\beta-1)(\beta-2) C' M_P^\beta (t - t_m)^{(\beta-3)} \\
&\quad - \gamma(\gamma-1)(\gamma-2) M_P^\gamma |t_s - t|^{(\gamma-3)}, \\
\dddot{H} &= \alpha(\alpha-1)(\alpha-2)(\alpha-3) B' M_P^\alpha t^{(\alpha-4)} + \beta(\beta-1)(\beta-2) \\
&\quad \times (\beta-3) C' M_P^\beta (t - t_m)^{(\beta-4)} + \gamma(\gamma-1)(\gamma-2)(\gamma-3) M_P^\gamma |t_s - t|^{(\gamma-4)}.
\end{aligned}$$

$$(19a, b, c, d)$$

If $a(t)$, given by eq.(18a), satisfies eqs.(12a) and (12b), ρ and p are obtained as

$$\begin{aligned}
\rho &= \frac{3H^2}{8\pi G} - \rho_A \\
&= \frac{3H^2}{8\pi G} - \frac{N}{180(4\pi)^2}[3H^4 + 6H\ddot{H} + 18H^2\dot{H} - 3\dot{H}^2] \\
p &= -\frac{1}{4\pi G}(\dot{H} + \frac{3}{2}H^2) + \frac{N}{360(4\pi)^2}(6H^4 + 8H^2\dot{H}) \\
&\quad + \frac{N}{180(4\pi)^2}[2\dddot{H} + 12H\ddot{H} + 18H^2\dot{H} + 9\dot{H}^2],
\end{aligned}$$

$$(20a, b)$$

where $H, \dot{H}, \ddot{H}$ and $\dddot{H}$ are given by eqs.(19a,b,c,d).

Eqs.(19 a,b,c,d) and (20a,b) show that ρ and p can be finite at $t = t_s$, if $\gamma \geq 4$. So, here onwards, $\gamma = 4$ is set in equations. Moreover , without any harm to physics, α and β are also taken as $\alpha = \beta = 3$.

Thus $a(t)$, given by eq.(18a),is obtained as

$$a(t) = exp[B'(M_P t)^3 + C' M_P^3 (t - t_m)^3 + M_P^4 |t_s - t|^4], \qquad (21a)$$

where $t > t_m$. If $a(t)$, given by eq.(22a), acquires its minimum a_s at $t = t_s, \dot{a}(t_s) = 0$ yielding

$$C' = -\frac{\alpha}{\beta} M_P^{(\alpha-\beta)} \frac{t_s^2}{(t_s - t_m)^2} B'. \qquad (21b)$$

Moreover, eq.(18b) is re-written as

$$B' = \frac{4 M_P |t_s - t_m|^2}{3 t_m^2}. \qquad (21c)$$

Use of $\alpha = \beta = 3$ and $\gamma = 4$, in eq.(19a), yields $H < 0$ implying $\dot{a}(t) < o$ during the period $t_m < t < t_s$. It shows contraction of the universe during this period. For $t > t_s$, eq.(19a) yields

$$H = 4 M_P^4 |t_s - t_m|^3 \Big(\frac{t_s}{t_m}\Big)^2 \Big[\Big(\frac{t}{t_m}\Big)^2 - \Big(\frac{[t - t_m]}{[t_s - t_m]}\Big)^2 + \Big(\frac{t_s}{t_m}\Big)^2 \Big(\frac{|t_s - t|}{|t_s - t_m|}\Big)^3\Big] > 0 \qquad (22)$$

showing expansion . At $t = t_s$, eq.(21a) yields

$$a(t_s) = exp\Big[\frac{4}{3} M_P^4 \frac{t_s^2}{t_m} (t_s - t_m)^3\Big]. \qquad (24)$$

Thus, it is obtained that, during the period $t_m < t < t_s$, universe will contract and will acquire its minimum at $t = t_s$. Later on, it will expand for $t > t_s$ showing a bounce at $t = t_s$.

As $H(t_s) = \dot{a}(t_s)/a(t_s) = 0$, eqs.(13) and (19 a,b,c,d) yield

$$\rho(t_s) = \frac{N}{15\pi^2} M_P^8 (t_s - t_m)^4 \Big(\frac{t_s}{t_m}\Big)^2 \qquad (24a)$$

and

$$p(t_s) = \frac{N}{60\pi^2}M_P^4 + \frac{2M_P^6}{\pi}(t_s - t_m)^2\Big(\frac{t_s}{t_m}\Big) + \frac{N}{4800\pi^4}M_P^8(t_s - t_m)^4\Big(\frac{t_s}{t_m}\Big)^2. \tag{24b}$$

RS II brane-world model

Classical approach

It is mentioned above that curvature invariants in future universe, dominated by phantom dark energy, get stronger with increasing time. So , relevance of high energy physics increases [12]. With this view, above type of analysis is done in brane-world also. Here, RSII model is considered due to its simplicity and geometric appeal. Moreover, it is more relevant to cosmology compared to RSI model. Modified version of eqs.(2a,b), for brane-gravity, are written as [6, 13]

$$H^2 = \frac{8\pi G}{3}\rho_{RS}\Big[1 + \frac{\rho_{RS}}{2\lambda}\Big] \tag{25a}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\Big[\rho_{RS} + 3p_{RS} + \frac{1}{\lambda}\rho_{RS}(2\rho_{RS} + 3p_{RS})\Big], \tag{25b}$$

where λ is the brane-tension.

Eq.(25a) yields

$$\rho_{RS} = \lambda\Big[-1 \pm \sqrt{1 + \frac{3M_P^2H^2}{16\pi\lambda}}\Big]. \tag{26}$$

Eqs.(3c) and (26) yield

$$\dot{\phi} = \omega^2\Big[-H\phi \pm \sqrt{(1-\omega^2)H^2\phi^2 + 2\omega^2\lambda\Big[-1 \pm \sqrt{1 + \frac{3M_P^2H^2}{4\pi\lambda}}\Big]}\Big], \tag{27}$$

In this model also, matter field is given by the same scalar field ϕ, so for ρ_{RS} and p_{RS} also eqs.(3c) and (3d) are used in eq.(25b). As a result, it is obtained that

$$-\frac{1}{4\pi G}(\dot{H}+3H^2) = \Big\{\Big(\frac{\omega^2}{2}-\frac{1}{3}\Big)\Big[-1\pm\sqrt{1+\frac{3M_P^2H^2}{16\pi\lambda}}\Big]-\frac{1}{3}\Big\}\times\Big[-H\phi$$
$$\pm\sqrt{(1-\omega^2)H^2\phi^2+2\lambda\omega^2\Big[-1\pm\sqrt{1+\frac{3M_P^2H^2}{16\pi\lambda}}\Big]}\Big]^2$$
$$-\omega^2\Big[-H^2\phi^2\pm\sqrt{(1-\omega^2)H^2\phi^2+2\lambda\omega^2\Big[-1\pm\sqrt{1+\frac{3M_P^2H^2}{16\pi\lambda}}\Big]}\Big]$$
$$-\phi^2\Big[-\frac{\dot{H}}{3}+H^2+\Big\{\frac{\omega^2}{3}(\dot{H}+H^2)-\Big(\frac{\dot{H}}{3}+\frac{H^2}{2}\Big)\Big\}\Big[[-1\pm\sqrt{1+\frac{3M_P^2H^2}{16\pi\lambda}}\Big]\Big]. \tag{28}$$

$a(t)$, given by eq.(5a), acquires its maximum at $t=t_m$, so $\dot{a}(t_m)=0$, but $\ddot{a}(t_m)\neq 0$. It yields $\dot{H}(t_m)\neq 0$. In case, $\omega^2=1$, eq.(28) implies that either $\dot{H}$ vanishes or it is complex. So, like GR-based model , $a(t)$ does not satisfy modified Friedmann equations (25a,b) for RSII model too, when $\omega^2=1$. But, in case of $\omega^2=-1$, $a(t)$, given by eq.(5a) satisfies eqs.(25a,b) and at $t=t_m, H(t_m)=0$ but $\dot{H}(t_m)\neq 0$ and

$$\phi^2_{RS}(t_m)=\frac{3}{4\pi G(1+\lambda)}. \tag{29}$$

Thus, like GR-model in RSII-model too, the scale factor $a(t)$, given by eq.(5a) satisfy eqs.(25a,b) for $\omega^2=-1$ (phantom conformal scalar), but not for $\omega^2=1$ (quintessence conformal scalar).

Quantum bounce near $t=t_s$

As $H(t_m)=0$, eq.(25a) yields

$$\rho_{RS}(t_m)=0, \tag{30a}$$

or

$$\rho_{RS}(t_m) = -2\lambda. \qquad (30b)$$

These results are obtained without making quantum correction in eq.(25a). Eq.(30a) corresponds to GR-model. Effect of the brane theory is realized in eq.(30b). So, for RSII model, only eq.(30b) is considered.

Like GR-model, here also quantum correction to eq.(25a) is made replacing $\rho_{(RS)}$ by $\rho_{(RS)} + \rho_A$ as

$$H^2 = \frac{8\pi G}{3}(\rho_{RS} + \rho_A)\Big[1 + \frac{(\rho_{RS} + \rho_A)}{2\lambda}\Big] \qquad (31)$$

Now using $H(t_m) = 0$, in eq.(31), it is obtained that

$$\rho_{RS}(t_m) = -2\lambda - \rho_A(t_m) = -2\lambda + \rho(t_m). \qquad (32)$$

where $\rho(t_m)$ is given by eq.(17).

It shows that, in RSII model, expansion of the universe stops at a lower energy density compared to GR case due to brane tension . Later on,like GR case, it contracts during the interval $t_m < t \leq t_s$ to its minimum $a(t_s)$ following $a(t)$, given by eq.(21a), with B', C' given by eqs.(21b) and (21c).

Like GR case, here also, at $t = t_s$ quantum correction to eq.(25a) leads to

$$\rho_{s(RS)} = -2\lambda + \rho(t_s) \qquad (33)$$

with $\rho(t_s)$, given by eq.(24a) .

Thus, in this case too, bounce will take place at $t = t_s$ like GR-model. In [14], bounce has been discussed using self-gravitational corrections to Friedmann equation for the brane cosmology,

Gauss-Bonnet model

Classical approach

Einstein-Gauss-Bonnet theory is a generalization over brane-gravity. So, it is reasonable to analyze this scenarion in in GB-model also.

Modified Friedmann equations for Einstein- Gauss- Bonnet theory is given as

$$H^2 = \frac{1}{4\tilde{\alpha}}\Big[(1-4\tilde{\alpha}\mu^2)cosh\Big(\frac{2\chi}{3}\Big) - 1\Big], \tag{34}$$

where $\tilde{\alpha} = \frac{1}{8g_s^2}$ (g_s is the string energy scale), energy density $\rho_{(GB)}$ and brane-tension $\lambda_{(GB)}$ is connected to energy parameter χ as

$$\begin{aligned} k_5^2(\rho_{(GB)} + \lambda_{(GB)}) &= \Big[\frac{2(1-4\tilde{\alpha}\mu^2)^3}{\tilde{\alpha}}\Big]^{1/2} sinh\chi \\ &= 2\sqrt{\mu^2 + H^2}[3 - 4\tilde{\alpha}\mu^2 + 8\tilde{\alpha}H^2] \\ &= 2\sqrt{\mu^2 + H^2}[k_5^2(2\mu)^{-1}\lambda^{\text{cri.}}_{(GB)} + 8\tilde{\alpha}H^2], \end{aligned} \tag{35a}$$

where

$$k_5^2\lambda^{\text{cri.}}_{(GB)} = 2\mu(3 - 4\tilde{\alpha}\mu^2), \tag{35b}$$

$$k_5^2 = \frac{(1+4\tilde{\alpha}\mu^2)}{\mu}k_4^2 = \frac{8\pi}{M_5^3} \tag{35c}$$

and

$$k_4^2 = \frac{8\pi}{M_P^2}. \tag{35d}$$

$\lambda^{\text{cri.}}_{(GB)}$ is obtained using the condition $\rho_{(GB)} = H = 0$ [15, 16]in eq.(35a).

Like RSII model, here also, matter field is given by the conformal scalar field ϕ used above. So, $\rho_{(GB)}$ and $p_{(GB)}$ are given by eqs.(3c,d). In GB-model, conservation equation looks like

$$\dot{\rho}_{(GB)} + 3H(\rho_{(GB)} + p_{(GB)} + \lambda_{(GB)}) = 0. \tag{36}$$

Connecting eqs.(35a,b,c,d) and (36), it is obtained that

$$\begin{aligned}\dot{H} &= -\frac{k_4^2 k_5^2\sqrt{\mu^2+H^2}}{2[k_5^2\mu^2+8\tilde{\alpha}k_4^2H^2]}(\rho_{(GB)}+p_{(GB)}+\lambda_{(GB)}) \\ &= -\frac{k_4^2 k_5^2\sqrt{\mu^2+H^2}}{2[\mu k_5^2+8\tilde{\alpha}k_4^2H^2]}\Big[(1-\frac{\omega^2}{3})\dot{\phi}^2+\{\frac{1}{3}(\omega^2-1)\dot{H}+\frac{\omega^2}{3}H^2\}\phi^2 \\ &\quad +H\phi\dot{\phi}+\lambda_{GB}\Big].\end{aligned} \tag{37}$$

Eqs.(3c) and (33a) yield

$$\dot{\phi}=\omega^2\Big[-H\phi\pm\sqrt{(1-\omega^2)H^2\phi^2+\frac{4\omega^2}{k_5^2}\sqrt{\mu^2+H^2}[3-4\tilde{\alpha}\mu^2+8\tilde{\alpha}H^2]-2\omega^2\lambda}\Big]. \tag{38}$$

At $H(t_m)=0$, using eqs.(3b,c,d), (35b) and (38) in eq.(37), it is obtained that

$$\Big[1+\frac{k_4^2}{6}(\omega^2-1)\phi^2\Big]\dot{H}(t_m)=-\frac{k_4^2}{2}\Big[2(1-\frac{\omega^2}{3})(\lambda_{GB}^{\text{cri.}}-\lambda_{GB})+\lambda_{GB}\Big]. \tag{39}$$

Case 1. For quintessence conformal scalar ϕ i.e. $\omega^2=1$

In this case, eq.(39) reduces to

$$\dot{H}(t_m)=-\frac{k_4^2}{2}\Big[\frac{4}{3}(\lambda_{GB}^{\text{cri.}}-\lambda_{GB})+\lambda_{GB}\Big]. \tag{40}$$

It shows that unless $\lambda_{GB}=4\lambda_{GB}^{\text{cri.}}$, $\dot{H}(t_m)$ does not vanish, so barring this particular situation, in GB-model, the scale factor given by eq.(5a) is valid for quintessence conformal scalar too. It is unlike the case of GR-model as well as RSII-model.

Case 2. For phantom conformal scalar ϕ i.e. $\omega^2=-1$

For this case, eq.(39) is obtained as

$$\Big[1-\frac{k_4^2}{3}\phi^2(t_m)\Big]\dot{H}(t_m)=-\frac{k_4^2}{2}\Big[\frac{4}{3}(\lambda_{GB}^{\text{cri.}}-\lambda_{GB})+\lambda_{GB}\Big]. \tag{41}$$

This equation shows that $\dot{H}(t_m)$ does not exist when $\phi^2(t_m)=3/k_4^2$. Moreover $\dot{H}(t_m)$ vanishes when $\lambda_{GB}=(8/5)\lambda_{GB}^{\text{cri.}}$ and $\phi^2(t_m)\neq 3/k_4^2$.

Thus, the scale factor $a(t)$ given by eq.(5a) is valid for phantom conformal scalar ϕ like GR-model as well as RSII-model, barring the situation when $\phi^2(t_m) = 3/k_4^2$ and $\lambda_{GB} = (8/5)\lambda_{GB}^{\text{cri.}}$

Quantum bounce near $t = t_s$

If expansion stops at $t = t_m, H(t_m) = 0$, so eq.(35a) yields

$$\rho_{GB}(t_m) = \lambda_{GB}^{\text{cri.}} - \lambda_{GB}. \tag{42}$$

It corresponds to GR-model in case $\lambda_{GB} = \lambda_{GB}^{\text{cri.}}$. Quantum correction to eq.(35a) modifies the equation (42) as

$$\begin{aligned} \rho_{GB}(t_m) &= \lambda_{GB}^{\text{cri.}} - \lambda_{GB} - \rho_A(t_m) \\ &= \lambda_{GB}^{\text{cri.}} - \lambda_{GB} + \rho(t_m) \end{aligned} \tag{43}$$

where $\rho(t_m)$ is given by eq.(17) for GR-model. This equation shows that $\rho_{\text{GB}}(t_m) > \rho(t_m)$, when $\lambda_{\text{GB}}^{\text{cri.}} > \lambda_{\text{GB}}$, otherwise $\rho_{\text{GB}}(t_m) < \rho(t_m)$.

Further universe contracts following the scale factor given by eq.(21a) and bounces at $t = t_s$. At $t = t_s$, the energy density is obtained as

$$\rho_{GB}(t_s) = \lambda_{GB}^{\text{cri.}} - \lambda_{GB} + \rho(t_s), \tag{44}$$

where $\rho(t_s)$ is given by eq.(24a).

Conclusion

In the cosmic picture explored above, it is found that quantum gravity makes drastic changes in the course of future accelerated universe driven by phantom dark energy in GR-model and RSII-model. But ,in case of GB-model, similar results are obtained for both quintessence dark energy and phantom dark energy. It is demonstrated above that, under quantum effects, universe expands upto a maximum when energy

density grows to $\rho(t_m), \rho_{RS}(t_m)$, and $\rho_{GB}(t_m)$, given by eqs.(17), (32) and (43) in GR , RSII and GB- models respectively. It is found that, in RSII model,due to brane-tension, $\rho_{RS}(t_m)$ is lower than the same in GR model. In GB-model, $\rho_{GB}(t_m)$ is lower than the same in GR model if brane-tension is higher than the critical brane-tension, but for brane-tension lower than the critical brane-tension $\rho_{GB}(t_m)$ is higher than $\rho(t_m)$.

Later on, it contracts upto a minimum size a_s at $t = t_s$, when energy density rises to ρ_s, given by eqs.(24a), (33) and (44) for GR , RSII and GB- models respectively. Like $\rho_{RS}(t_m)$, $\rho_{RS}(t_s)$ is also lower in RSII-model. In GB-model, $\rho_{GB}(t_s)$ is lower or higher than $\rho(t_s)$ depending on the brane-tension like $\rho_{GB}(t_m)$. Finite energy density is obtained when $\alpha = \beta = 3$ and $\gamma = 4$ in the scale factor $a(t)$ given by eq.(18a). It shows that universe will contract during the time period $t_m < t \leq t_s$ and expand for $t > t_s$ exhibiting a bounce at $t = t_s$. Thus, a possibility of re-birth to an exponentially expanding non-singular universe for $t > t_s$ is obtained. In the present cosmic picture, *future singularity* is not mild, but cosmic bounce at $t = t_s$ shows its complete avoidance.

References

[1] Riess, A. G. *et al.*, Astron. J. **116**,1009 (1998) ; astro-ph/9805201; Tonry, J.L. *et al.*, astro-ph/0305008; Garnavich, P. *et al.*, Astrophys. J. Lett. **493**,L53 (1998) ; Schmidt,B.P. *et al.*, *ibid* **507**,46 (1998) ; Bahcall, N.A. *et al.*, Science, **284**,1481 (1991) ; Perlmutter , S. J.*et al.*, Astrophys. J. **517**,565 (1999); astro-ph/9812133; Spergel, D. N. *et al.*, Astrophys J. Suppl. **148**,175 (2003)[astro-ph/0302209].

[2] Riess, A. G.*et al.* , Astrophys. J. **607**,665 (2004) [astro-ph/0402512].

[3] Caldwell, R.R. , Phys. Lett. B **545**, 23 (2002) ; Caldwell,R.R.,Kamionkowski, M., Weinberg, N.N. , Phys. Rev. Lett. **91**, 071301 (2003) ; Daly, R.A. *et al.*, astro-ph/0203113; Daly, R.A. Daly , Guerra, E.J. , Astron. J. **124**, 1831 (2002) ; Daly, R.A. , astro-ph/0212107; Hannestad, S., Mortsell, E. , Phys. Rev.D**66**, 063508 (2002) ; Melchiorri,A. *et al.*, Phys. Rev.D**68**, 043509 (2003)

; Schuecker, P. *et al.*, astro-ph/0211480; Srivastava, S.K., Phys. Lett. **619 B**, 1 (2005) [astro-ph/0407048].

[4] Copeland, E.J.,Sami, M., Tsujikawa, S. ; hep-th/0603057.

[5] Elizalde, E., Nojiri, S., Odintsov, S.D. Odintsov, Phys. Rev.D**70**, 043539 (2004).

[6] Nojiri, S., Odintsov, S.D. Odintsov, hep-th/0405078; Phys.Lett. **B 595**,1 (2004); hep-th/0408170.

[7] Szydlowski, M. , Krawiee, A., Czaja, W. , Phys.Rev.E**72**, 036221 (2005) [astro-ph/0401293].

[8] Miller, A.D. *et al.* , Astrophys. J. Lett. **524**, L1 (1999) ; de Bernadis, P. *et al.* , Nature (London)**400**, 955 (2000) ; Lange, A.E. *et al.* , Phys. Rev.D**63**, 042001 (2001) ; Melchiorri, A. *et al.* , Astrophys. J. Lett. **536**, L63(2000) L63; Hanay, S. *et al.* , Astrophys. J. Lett. **545**, L5 (2000) .

[9] Faraoni, V., Phys. Rev.D**68**, 063508 (2003) .

[10] Dabrowski,M.P., Phys. Rev.D**71**, 103505 (2005); Balcerzak, A., Dabrowski,M.P., Phys. Rev.D**73**, 101301(R) (2006).

[11] Birrel, N.D., Davies, P.C.W., Quantum Fields in Curved Space (Cambridge Univ. Press, Cambridge,1982).

[12] Brown, M. G., Freese, K. ; astro-ph/0405353.

[13] Maartens, R. , Living Rev. Rel.**7** ,7 (2004); gr-qc/0312059.

[14] Setare, M.R., Phys.Lett.B **602**, 1 (2004); Phys.Lett.B **612**, 100 (2005).

[15] Sami, M., Savchenko, N., Toporensky, A. ,Phys. Rev.D **70**, 123528 (2004); hep-th/0408140.

[16] Dufaux, Jean-Francois, Lidsey, James E. ,Maartens, R., Sami, M., Phys. Rev.D **70**, 083525 (2004).